+22

Commentary

Saturn, I

1. **(4 days)** Students can draw a diagram for the worm's trip. The first day, he reaches 4.5 feet but then slips back to 2 feet level at night. The next day he reaches 6.5 feet, but then slips back to 4 feet at night. The 3rd day he reaches 8.5 feet but then slips back to 6 feet. On the fourth day, he reaches 10 feet and is on top of the hill, so doesn't slip back.

2. **(f)** Problems such as this one should help students realize that their answer should make sense in terms of the real world. Knowing that a soda costs around 60¢, the challenge for the student is to decide which of these answers is the correct way to interpret the calculator display. This problem might lead to a class discussion about common misuse of the decimal point in advertising, such as writing ".60¢" for "60¢" and "$199" for "$1.99."

3. **(Thursday)** Students might list the days of the week, and count from the 9th starting on a Tuesday.

4. $\left(7\overline{\smash{)}\mathbf{301}}\right)$ This problem can be approached through *guess-check-revise.*

5. **(Either the top and bottom can be circled, or the two sides)** A set of parallel lines are lines that never cross. These can be demonstrated by having students use pieces of spaghetti to represent lines. They might be encouraged to look for parallel lines immediately around them--notebook paper, the top and bottom of classroom walls, etc.

6. **(96 mm, may want to accept anything from 94 to 97 mm)** Students should use a metric ruler rather than one marked in inches. They might count each centimeter mark as 10 millimeters, and then the extra millimeters, from the eraser to the tip of the pencil.

7. **(10 ways)** Sugar, unifix or wooden cubes can be used for students to go through the experiment in a concrete way. The faces can be colored with a crayon or magic marker, or simply labelled "G" and "W." At home, students can use any box they can find although using only 1 box over-and-over means they must be careful in keeping track of the different cubes already made.

   Combinations: *1 cube each*--6 W; 6 G; 1 G and 5 W; 5G and 1W

   2 ways each--2 G and 4 W; 2 W and 4 G; 3 G and 3 W

8. **(NO)** Students should realize a milk shake costs more than $0.39, unless there's a special. If a student mentions this, they should receive credit also.

9. **(85, 67, 49)** Students might list all the pairs of single digits with a sum of 13. Each such pair of digits make up 2 two-digit numbers. Only the resulting numbers with the odd digit in the units place is not divisible by 2.

   8 + 5 = 13, giving 85 as a solution (but not 58)
   6 + 7 = 13, giving 67 as a solution (but not 76)
   4 + 9 = 13, giving 49 as a solution (but not 94)

20

Commentary

Saturn, II

1. **[(8 ÷ 4) + (6 x 2) = 14]** Students can use *trial and error* to find the correct order.

2. **(30 squares)** There are 16 small squares, 9 of the next largest in which 4 of the smallest are put together, 4 of the next largest of 9 small ones together, and the one large square itself.

3. **(left-hand calculator)** Students who have trouble with this problem might be encouraged to think of money. The 0.4 might be $\frac{4}{10}$ of a dollar or 40¢, whereas 0.39 might be 39¢. Another way would be for a student to subtract each number from the other on a calculator. The way which gives a positive number on the display means the largest number was entered first.

4. **(4)** Students might divide the total number of people going by the number of people that can fit in one van, with one person per seat belt. If so, they should realize that 21 people can go in 3 vans, but an extra van is needed for the remaining 4 people. This is a case in which the answer to a division problem requires rounding the decimal remainder up, rather than to the nearest whole number.

5. **(9708.6)** This is a simple recognition of place value task.

6. **(1 kg; 350 mL; 30° C; 2200 km)** Students should be encouraged to use "bench mark" metric measurements to estimate reasonable answers. For example, their math book weighs about a kilogram, a mL is about one drop from an eyedropper, a comfortable room temperature is about 30° C, and the distance across the United States is about 5000 km.

7. **(3 to 10, 3:10, or $\frac{3}{10}$; 4 to 15, 4:15, or $\frac{4}{15}$; boys)** Any of these answer forms are acceptable. To find which ratio is larger, 3:10 or 4:15, students can be encouraged to transform the ratios by doubling, tripling, etc., until they get two ratios with the same size comparison group. By doubling, 3 boys out of 10 is the same as 6 boys out of 20. By tripling, you get the ratio 9 boys out of 30. By doubling, 4 girls out of 15 is the same as 8 out of 30. Since 9 out of 30 is more than 8 out of 30, the boys with braces represents a larger ratio.

8. **($2.75 total cost and $2.25 change)** Sales tax of 6% can be interpreted by students as paying $.06 on each dollar spent. On $2 spent, the tax would be $0.12 and on 59¢, the tax would be another 4¢. Sales tax is a real-life example in which partial amounts of money are *rounded up*, rather than to the *nearest,* penny. The total cost would then be $2.59 + 12¢ + 4¢ or $2.75; the change from $5 would then be $2.25.

16

Commentary

Saturn, III

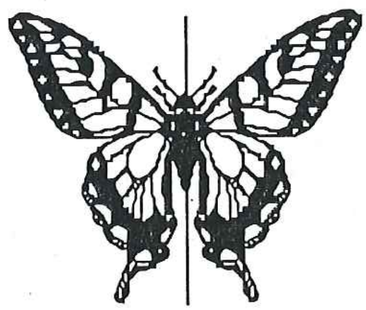
1. **($159.18)** Multiply 36 times $2.38 and 42 times $1.75. Add the two totals together.

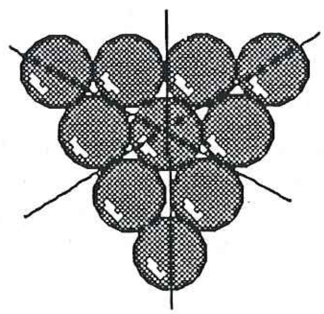
2. **(6 students)** Students can count the number of students for each grade, adding grades 1-3 together and grades 4-5 together, and subtract to find the difference. Or they might count the total number of stars in each group, subtract and find a difference of 2 stars, then multiply by 3.

3. **($15)** Students might find $\frac{1}{4}$ of $100, getting $25. They have $75 left. They can find $\frac{1}{5}$ of $75 by dividing 75 into five equal shares, getting $15 per share. That's the amount saved.

4. **[(8÷4) – (2÷1) is one possibility.]** This is a *guess-check-revise* problem. They must substitute until they come up with the correct order.

5. **Answers shown below.**





6. **Answers will vary.** To decide if the figure is symmetric about the line, fold it and see if the sides match up.

7. **(400)** Students might make a list to organize the approach to this problem. Such a list as the one below helps to observe a pattern:

Group number	People in group	Total
1	1	1
2	3	4
3	5	9
4	7	16
.	.	.
.	.	.
20	39	

If students don't notice the pattern that the total after n groups is n^2, they can still solve the problem by adding the "people in each group" column. Notice that 1+39 = 40, 3+37=40, 5+35=40, etc. There are ten such subtotals of 40, giving 400.

8. **(6)** Students might find this in a variety of ways. One way is to look at the third scale and conclude that 1 turtle weighs 0.5, and then from the second scale the 2 turtles would weigh 1 leaving the cake to weigh 12. Then from the first scale, you know that the can must weigh 6 since the cake weighs 12.

17

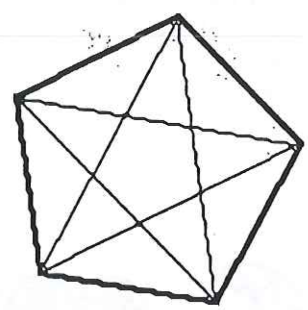
Commentary

Saturn, IV

1. **(10, 15, 21, 28, 36)** Triangular numbers can be found by arranging a number of dots in a pattern , and the pattern forms an equilateral triangle.

2. **($24.00 more)** $5.25 x 40 = $210, $5.85 x 40 = $234, and $234.00 – 210.00 = $24.00. Another approach would be to notice that there's a difference of 60¢ in the hourly rate, and 40 x $0.60 = $24.

3. **(5 diagonals)**



4. **(One fourth of the dollar bill should be shaded. $1.25)** Students can shade in 1/4 of the dollar bill in several different ways. The only criteria is that the dollar bill be divided into 4 equal pieces, and 1 is shaded. Hopefully students will realize that 1/4 of the dollar bill is equivalent, money-wise, to a quarter. This will enable them to find the answer to the second part.

5. **(8 meters and 4 meters)** The students will guess all the pairs of numbers that can be multiplied together to give 32. Then they have to see if the pairs can be added to get a perimeter of 24. Drawing a picture helps.

6. **(50)** This problem is a precursor to solving an equation of the form $3Y + 31 = 181$. Students intuitively know that they can remove the 31 from the scale, and have 3 cans that weigh 150. Dividing 150 by 3 gives that each can weighs 50. So $Y = 50$ solves the equation.

7. **(7 x 12 = 84 or 12 x 7 = 84; 39 x 2 = 78 or 2 x 39 = 78; (20 + 3) x 4 = 92 or (3 + 20) x 4 = 92 or 4 x 23 + 0 = 92)** The problems can be found by using number sense, and estimating mentally. Some students might not use parentheses in writing the last problem. In fact, on most calculators it's not necessary to use parentheses for this problem. However, writing the problem out to be done by hand requires parentheses for without it, *order of operations* would require multiplying 3 x 4 first, and adding that to 20, resulting in 32.

8. **(37)** The numerical pattern for the number of squares is: 1, 5, 9, 13, 17, ... Adding 4 more squares to each figure produces the next figure. Algebraically, if the figure number is n, the number of squares could be written as $4n - 3$.

20

Commentary

Saturn, V

1. ($\frac{9}{16}$; $\frac{1}{4}$) This is a real world example where students need to find a common denominator to be able to do the problem mathematically. It might be nice to bring such a set of wrenches so that when students turn in their paper, they can actually compare their results with what the wrenches tell them. Some students might solve this problem using real wrenches at home, avoiding the mathematics altogether.

2. (**$12.72**) Find a third of $18, and then subtract that amount to leave $12. Six percent of $12 is $0.72, which is added. Another way to approach the problem is to multiply $12 by 1.06, which gives the total cost, including sales tax.

3. (**1770**) Students can compute 42 x 35 and add that to 12 x 25.

4. (**45**) A corner of a sheet of paper can be placed over the hole where the piece of pizza is being removed. It's easy to see that this hole is about half of the square corner.

5. (**May; 19.05**) For students to be successful, they need to understand that zeros can be added to the right end of a decimal without changing its value. Therefore 9.6 can be thought of as 9.60, and 9.60 is easy to compare with 9.45. It's also easy to add the two, once they have the same number of decimal places.

6. (**4 yd. 2 ft. 2 in.**) Add and get a total of inches, feet and yards. Then convert each measurement to the next highest measurement.

7. (**15**) Candles can be counted in groups of 3. Each group of three candles represents 5 years. The nine candles on the cake give three groups of 3, which corresponds to 15 years.

8. (**d. Justin**) Make a chart. The chart could have four columns across and four down. The top could be labeled gray, green, blue, and white. The side could then be labeled Tia, Matt, Kenya, and Justin. Eliminate things that can't be true, resulting in the final choice.

9. (**3 is circled; 9, 13, 17, 21, 25 or any number 1 more than a multiple of 4; 4**) Hopefully students will notice as they count to find the answers that there is a numerical pattern that underlies these figures. They repeat every four figures, so number 4 will always be like the other multiples of 4 in the pattern. Number 1 will be like the multiples of four plus 1, and so on.

21

Commentary

Saturn, VI

1. ($\frac{1}{4}$ **or 25%,** $\frac{3}{4}$ **or 75%**) There are four numbers on the spinner. Therefore, the chances of getting 4 is one out of four or 1/4. The chances of not getting a 4 is 3 out of 4, or 3/4. These could also be written as percentages.

2. **(6¢, 15¢, 24¢, 33¢, 42¢, 51¢, and 60¢)** Students can make a chart or list of the possible combinations of coins that would fit the criteria. A chart like the one below might be made:

pennies	6	5	4	3	2	1	0
dimes	0	1	2	3	4	5	6
money	6¢	15¢	24¢	33¢	42¢	51¢	60¢

3. **(80)** Computing inside the parentheses is important. The problem is written so that students can use number sense to compute inside the parentheses easily -- 7.5 is $7\frac{1}{2}$, and $7\frac{1}{2} + 2\frac{1}{2}$ gives 10. Then 8 x 10 is 80.

4. **(Not enough information -- you need to know the cost to mail the sweatshirt.)**

5. **(Measure the student's line. It should be 52 mm.)**

6. **(850; 150; 450)** For (a), find 350 + 300 + 200 or 850. For (b), compute 350 - 200 to get 150. To find (c), add 350 and 300 to get 650, then subtract 200 to get 450.

7. **(2)** This problem can lead to algebraic thinking. A variable *d* is introduced, along with a diagram that students can use to find the value of the variable. They can *guess-check-revise* to find *d*, or solve the situation logically as they would the equation $4d + 3 = 11$, by subtracting 3 from 11 and then dividing what's left by 4. The problem is intended to help students see a real-life situation that would later lead to an equation, and know that in such cases their solution to the equation should make sense in the real world.

8. **(Thursday)** Students can tell from the graph that the total distance did not change on Thursday, because the line was horizontal at that point. Therefore that's the day when she did not ride her bike to school.

9. **(a. 1,020; b. 782)** For (a), multiply the highest number of students per class by 34. For (b), multiply the lowest number of students per class by 34.

24

Commentary

Saturn, VII

1. **($31\frac{1}{12}$)** Students will need to find a common denominator for the fractions. 12 is the smallest such, although others (24, 36, etc.) would work also. If the fractions are converted into those with denominator 12, they will sum to 25/12 or $2\frac{1}{12}$. When added to the whole number parts, the answer is $31\frac{1}{12}$.

2. **($27.60)** Students can find one-fifth of $34.50 by dividing by 5. They then subtract this from $34.50. Another way would be to find four-fifths (or 80%) of $34.50.

3. **($9.34 and $10.75)** Students will have to use their visual acuity to see the sides of the figures that aren't shown. The top figure has two square faces at $1.49 each, and four rectangular faces at $1.59 each. It's price is given by: ($1.49 x 2) + ($1.59 x 4) = $9.34. The other figure has two triangular pieces at $2.99 each, and three rectangular pieces at $1.59 each. It's total price is given by ($2.99 x 2) + ($1.59 x 3) = $10.75.

4. **(a. > b. = c. > d. <)** In (a), the students can change 1/2 to 0.50 and compare 34.63 to 34.50. In (b), students can think of 1 as 5/5, so by taking 2 whole units from $3\frac{2}{5}$ and changing them into fifths, they would get 10/5. Or, $3\frac{2}{5} = 2\frac{7}{5} = 1\frac{12}{5}$. In (d), students have to realize that 9/100 is smaller than 9/10.

5. **(5 hours 11 minutes)** Count the hours from 8:15 to 1:15, then the minutes from 1:15 to 1:26.

6. **(a. 1,000,000; b. 10,000; c. 1,000)** This is a good problem to check on *number sense* for students. Students who have trouble with (b) and (c) might profit from starting with smaller numbers in similar problems.

7. **(E = 2; F = 1; G = 7; H = 8)** Students might start by noticing several critical features of this problem. E must be either 1 or 2, since the answer does not carry over into the ten thousands place. They might further guess that E ≠ 1 since this would result in H = 4, and the problem doesn't disallow this, but 4 is already in use so it's not likely. Choose E = 2 and assume H = 8, then, and proceed from there.

8. **(26.46)** Multiplication is called for to find the area of the carpet. 6.3 x 4.2 = 26.46

9. **(How do you keep a turkey in suspense?)** This riddle is a fun way for students to practice finding a fractional part of a set. Some possible answers are "I'll tell you tomorrow!" and "Delay Thanksgiving one day!"

20

Commentary

Saturn, VIII

1. **(a. true, b. false, c. true)** Perpendicular lines intersect and form right angles. Parallel lines do not cross or intersect. Students can draw diagrams or work with spaghetti to see if these statements seem true to them. For the last one, they might consider the lines on a sheet of notebook paper, for verification.

2. **(6)** This problem can verify if students can use *order of operations*. Work the parentheses first, divide, then add. Notice that if students do this problem left to right, as if entering it in a calculator, they would get the answer 20/3.

3. **(Lisa's stick, by 2 inches)** Lisa's stick is 2/3 of a yard, which is 2 feet. Sandy's is $1\frac{10}{12}$ feet, or 1 foot, 10 inches. 2 feet is longer than 1 foot, 10 inches by 2 inches.

4. ($\mathbf{\frac{3}{8}}$) The square can be divided into eight equal parts. If the square in the lower left corner were partitioned into two parts, three-eighths would be shaded.

5. **(10 hours, 13 minutes)** Adrienne traveled 5 hours, 28 minutes. Erica traveled 4 hours, 45 minutes. The only difficult part is to rename the total minutes, 73, as 1 hour, 13 minutes.

6. ($\mathbf{\frac{2}{18}}$ **or** $\mathbf{\frac{1}{9}}$) There are 18 jellybeans in the bag. Two of them are orange. The chances of pulling out an orange marble would be 2 out of 18. In lowest terms, the answer would be 1 out of 9.

7. **(4 + 3 -7 + [6 ÷ (10 ÷ 5)] is one possibility.)** There may be other solutions. Check each answer. Students may use parentheses.

8. **(2)** Joe gave away 2/3 of six colas, or 4 colas, leaving him 2. Christine gave away half as many as Joe, so she gave away 2 colas, leaving her with 4. So she had 2 more than Joe, in the end.

9. **(5)** The problem will show if some students mistakenly apply the traditional method of solving subtraction word problems -- "*how many left* means to subtract." In this case, the number left is the same as the number who couldn't squeeze into the refreshment stand.

+ 17

Commentary

Saturn, IX

1. **(2 quarters, 3 dimes, 1 nickel, 2 pennies or 1 half dollar, 2 dimes, 3 nickels, 2 pennies)** Students can experiment with coin values to find the answer. It helps to write down some headings -- half dollars, quarters, dimes, nickels, pennies -- and begin listing coins under them that sum to 87¢, checking to see if you have eight coins. If not, modify the list. Notice that right away, you can tell that you have to have at least two pennies.

2. **(Yes, to the problem below.)** Write this problem on several 3 by 5 cards so students can read the problem privately, estimate, and write their answer down when they hand in their paper:

Martin has \$20. He wants to buy a magazine for \$3.95, a baseball cap for 5.99, and a cola for 89¢. Will he have enough left to spend \$6 on a movie ticket?

3. **(4 green, 2 blue, 3 white)** Finding the least common multiple will help students determine that Jack must buy 12 of each color ornament. An intuitive way for students to find the *least common multiple* is: Start with the largest number, 6, and look at its multiples, 6, 12, 18, and so on. When you find a number that's also a multiple of both other numbers, you've found the *least common multiple*.

4. **(vertical line down the middle, 8)** The "fold line" or *line of symmetry* splits the space ship in half, along the vertical. The area is found by counting 6 whole squares and 4 half squares, for a total are of 8.

5. **(The center number is 5. Numbers in "opposite boxes" total 10)** Students might solve this by guess and check, or they might think of what must be true for 3 numbers to sum to 15. Their average would have to be 5, so start by placing 5 in the center box. Then the other two numbers along each line have to total 10 for the whole line, including 5, to sum to 15. So just pick numbers for "opposite boxes" that sum to 10.

6. **(a. 11 million b. 8.5 million c. 16 million)** Answers may vary somewhat from these given, particularly (b), but they should be close to these numbers.

7. **(a. 3rd from left b. $\frac{3}{4}$ or 75% c. $\frac{1}{4}$ or 25%)** In this problem, the chances of winning are related to the area of the circular space. The white team's space is about $\frac{1}{4}$ of the area of the circle in the 1st and 2nd spinners, and about $\frac{1}{2}$ in the 4th spinner.

X16

Commentary

Saturn, X

1. **(474.25 or 474 1/4 feet, Flights--about 11)** The first part of this problem is simply averaging the four distances given -- 120, 585, 340, and 852. Students might have to look up the number of feet in a mile -- 5,280. They can then divide that number by their average and round off the answer.

2. **(39 or 40 feet)** The scale shows 10 feet. Measuring accurately gives 39.5 feet, so accept an answer anywhere between 39 and 40 feet. It might be interesting to extend the thinking by asking questions such as -- would this plane fit in your classroom? In your garage?

3. **(a. 324 times b. 81)** Students might first find 1/3 of 162 games, then double that amount for 2/3. That answer of 108 is then multiplied by 3, obtaining 324. Very few students will notice that 3 x 2/3 x 162 can be found by simply multiplying 2 x 162. For part (b), students can either find 1/4 of 324 by dividing by 4, or find 25% of 324 by multiplying 324 by 0.25.

4. **(a. 432 ft. b.$694.98)** Students might profit from drawing a sketch of the yard, and labeling the four sides with their lengths. The first answer is obtained by simply adding 96, 120, 96, and 120. The second can be found by dividing 432 by 8 and then multiplying by $12.87.

5. **(4)** It is possible to pull out one of each color marble on the first three draws. Therefore, the fourth marble will match one of the first three.

6. **($95)** Have the problem below written on several 3 by 5 cards for students to read prior to handing in their paper. They must do the problem in their heads, and simply write the answer in the space provided. Number sense will play a role here, as $18.95 is about $1 less than $20. So 5 times $18.95 should be close to 5 x $20, less $5.

> Chris needs to buy five new shirts for a vacation trip coming up over Thanksgiving. The shirts are on sale for $18.95 each. What is the best estimate of what the five shirts might cost?
>
> a. $75 b. $85 c. $95 d. $105

7. **(7)** Students should be encouraged to *guess-check-revise* to find the value of *X*. They might try *X* = 1 to start, and see that this results in less than 18 when used in the left side of the number sentence. So they would adjust their guess up, and continue until they found that *X* = 7 produces 18, when the left side is computed.

8. **(a. $1 b. $2.50 c. $1.50)** Snacks consume 20% of Danny's money, and 20% of $5 is 1/5 of $5, or $1. The graph is divided so that his savings are half of his money, and half of $5 is $2.50. The percent spent on entertainment can be found by adding 50% and 20% to get 70%, and realizing that the rest of the chart must then be 30%. The entertainment money is then 30% of $5, or $1.50.

9. **(a. 60 b. 80)** Drawing a picture might help students interpret what "3 boys for every 4 girls" means. They can put together two such groups and know that "6 boys for every 8 girls" is the same ratio, but with larger numbers. Continuing in this fashion, by using ten such groups, they would have the proper number overall -- 140 students, consisting of 60 boys and 80 girls.

Commentary

Saturn, XI

+21

1. **(Howard--Recorder, Jacqueline--Materials Manager, Billy--Time Keeper, Kanisha--Reporter)** Students might make a chart crossing out the jobs that each student does not have. From "Kanisha sits across from the Recorder and next to the Materials Manager," for example, we can mark off that Kanisha has neither of those jobs. A chart and *process of elimination* can therefore be used to match each job with the child.

2. **(a. 4; b. 32)** It would help for students to draw and label a diagram of the floor. For (a), they need a separate 2 by 4 for each 8-ft. side of the plywood sheet, and another to go down the middle. They can buy one more 8-ft. 2 by 4 and cut it in half to get two 4-ft lengths for the short sides of the plywood. This is a total of four 2 by 4's. For (b), the area of the plywood sheet is the amount of carpet to purchase--4 x 8 or 32 square feet.

3. **($90.20)** First, students need to be sure that tapes are cheaper if bought in packages of 3 than in packages of 2. Then the strategy of "buy all the packages of 3 you can first, and finish out with packages of 2" can be used. Seven packages of 3 tapes per package can be purchased for $82.25. Two more tapes are needed to total 23, and one package of 2 will add $7.95 for a total of $90.20.

4. **(80 gms.)** 2 spheres (240 gms) equal 6 boxes, so a box must equal 40 gms. If one sphere and one box (120 + 40) equal 2 pyramids, then each pyramid is half of that sum, or 80.

   Students might be encouraged to begin writing an explanation of how they solve these types of problems using a variable as shorthand notation. For example, they might show the steps above as:
   $2s = 240 = 6b$, so $1b = 40$ from the left scale.
   $1s + 1b = 120 + 40 = 160 = 2p$, so $1p = 80$ from the right scale.

5. **(1st day--54 cans, 2nd day--49 cans)** Students might estimate half of the cans collected as 50, and adjust that number up or down for the two days using *guess and test* to meet the conditions of the problem.

6. **($46.64)** Students with good number sense might think of 20% as $\frac{1}{5}$, and one-fifth of $55 is $11 off, so the sale price of the shoes is $44. Tax is $2.64. Another way to approach the problem, particularly if a calculator is handy, is to realize that she will pay 80% of the regular price and compute $55 x 80%. This amount is then multiplied by 1.06 to "add on" the sales tax in one step.

7. **(735)** It's interesting for students to realize that people who work mentally in arithmetic sometimes follow the "reverse procedure" from what they are taught to do with paper-and-pencil. In this case, James works with the larger numbers first, and works his way down to the smaller numbers. Notice that if he makes a mistake somewhere down the line, he'll probably be close to the right answer because he dealt with the larger number first.

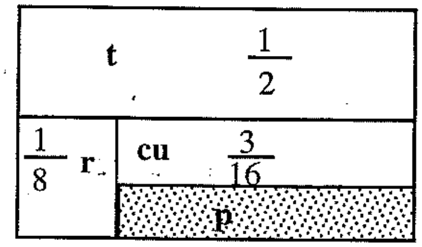
   Have this problem on 3 x 5 index cards for students to see when they hand in their paper. They are allowed to write only their answer down: 21 x 35.

8. **($\overline{FE}$ = 8 ft.; $\overline{CD}$ = 20 ft.; $\overline{BF}$ = 4 ft.)** Students can find $\overline{FE}$ by noticing that it's visually a little shorter than $\overline{AB}$. $\overline{CD}$ is exactly twice $\overline{AB}$. $\overline{BF}$ is less than half of $\overline{AB}$ visually, but not enough less to be 1.

20

Commentary

Saturn, XII



1. ($\frac{3}{16}$) Students are encouraged to make a diagram. If they do so, they can find the area of the peppers in several ways. The tomatoes take up $\frac{1}{2}$, and the radishes $\frac{1}{4}$ of what is left, or $\frac{1}{8}$ of the total garden. This leaves $\frac{3}{8}$ of the garden for the cucumbers and peppers, to be split evenly. Thinking of $\frac{3}{8}$ as $\frac{6}{16}$, it's easy to see that half of that is $\frac{3}{16}$.

2. **(In 1996, 293 years; in 1997, 294 years; etc.)** This subtraction problem can be enhanced with a little Florida history about St. Augustine being the oldest city in the United States.

3. **(6 outfits)** One strategy is to make a chart, as below. Another is to make a diagram.

	red shorts	white shorts	blue shorts
red shirt	4	4	4
white shirt	4	4	4

4. **(18)** One strategy is to work backwards, asking students what they would have if they had not subtracted 6. They should see that they would have 42. Therefore, something times 7 equals 42. Knowing that the something must be 6, they will recognize that 18 divided by 3 equals 6. Another approach is to *guess-check-revise*. They could start by guessing a number like 30 for *n*, and check to see that 30 is too high because [(30÷3) x 7 – 6] gives 64, not 36. So they would adjust the guess down, and try again.

5. **(D, or 7/10)** Students could place these numbers on a number line or divide a piece of paper into pieces for comparison. Another method would be to change them all so they have a common denominator (30 is the least such) and compare the resulting numerators.

6. **(a little less than 7 miles high)** 5,280 is a little more than 5,000. 5,000 goes into 35,000 exactly 7 times. So 5,280 would go into 35,000 a little less than 7 times.

7. **(4.5 mi.)** The 45-mile race would have no checkpoints at the start and finish lines. A picture will show that there are 9 checkpoints resulting in 10 spaces between "start" and "finish."

8. **($2\frac{1}{2}$ hours; increasing; constant; last half hour)** This problem has students look at a line graph and interpret it visually. The graph stops at about $2\frac{1}{2}$ hours along the time axis, meaning it took him about $2\frac{1}{2}$ hours to finish. The line is going up at a constant rate during the first half hour, so his speed is increasing.) During the second half hour, the line is horizontal (meaning his speed was constant. The line is at its highest point during the last half hour, meaning that during this time period he was traveling at his greatest speed. An extension activity would be for students to make their own graph of his trip, to match a story they make up about his speed, and to included his check-point stor

2 1

Commentary

Saturn, XIII

1. **(13)** Students might forget that the 9 remaining when 129 is divided by 10 represents seats which are needed for the class. Twelve rows would not be sufficient.

2. **(about 100 pounds per month)** Students will need to read carefully--the amount produced annually is given in tons, but the answer is asked for in pounds. Students can multiply tons of garbage per year by 2,000 to get pounds per year, then divide by 12 to get pounds per month. This result is then divided by 250 million to get each person's share per month. The actual answer is about 103 pounds per month, but any reasonable answer should be accepted.

3. **(1st row: 8, 1, 6; 2nd row: 3, 5, 7; 3rd row: 4, 9, 2)** There are several possible solutions to this magic square. One possibility is given. Students may be encouraged to use number tiles to help solve the problem. One strategy that would help is to assume the center number might be 5, which is the middle number of 1-9. Then you know that "opposite diagonal numbers" must sum to 10, so place 8, 6, 4, and 2 in the diagonals in this fashion. Continue making good guesses, checking, and making adjustments as called for.

4. **(25, 36, 49; yes; no)** Students can continue making figures by hand, if necessary, but hopefully they will notice a relationship between the number of the figure (1st, 2nd, 3rd, ...) and the total number of dots in the figure (1, 4, 9, ...). The next figure in the pattern would have 5 dots on each side, and so 25 in all, and so on. 100 is a square number because a 10-by-10 square can be formed from 100 dots. 200 is not a square number -- a 14-by-14 square would have 196 dots, and a 15-by-15 would have 225.

5. **(5)** The rectangles are: 1 x 36; 2 x 18, 3 x 12; 4 x 9; 6 x 6

6. **(c)** Students would enjoy experimenting with these three ways of earning money, using a calculator. (a) would give you only \$0.50 x 31 or \$15.50 for the month. (b) would give you the sum of the numbers from 1 to 31, multiplied by \$0.10, or \$49.60. (c) shows the power of doubling -- by the 15th day, for example, you would make \$163.84 on that day alone. Using the calculator, your group will notice the rapidity with which the product increases when the number doubles daily.

7. **($\frac{2}{4}$ or $\frac{1}{2}$; $\frac{1}{4}$)** Since the four aces have two red ones (hearts and diamonds), the chances of pulling a red card at random are 2 in 4 (written either $\frac{2}{4}$, $\frac{1}{2}$, or 50%). There is one club out of the four cards, so the chances of pulling a club at random from the bag is 1 in 4, written as $\frac{1}{4}$ or 25%.

8. **(far right card)** Students with good spatial visualization skills will find this card quite readily. Other students may profit from actually drawing the design on a thin sheet of paper, and follow the steps in order holding the paper up to the light to see what happens.

9. **(0)** Students with good operation sense will realize, if they "look ahead" in the problem, that the answer is zero. Any number multiplied by zero results in zero, so if zero is one of the factors shown in a multiplication problem such as this, the answer is automatically zero.

20

Commentary
Saturn, *XIV*

1. **(answer shown below)** These visual puzzles become increasingly difficult, but notice that some students will see the analogies immediately. In (a), the figure goes simply from unshaded to shaded. In (b), only two-thirds of the figure is considered, and it also goes from unshaded to shaded. In (c), the figure turns 180°.

2. **($\frac{1}{8}$)** Students might guess-check-revise to find the fraction, or they might notice that $\frac{4}{8} + \frac{5}{8}$ will give $\frac{9}{8}$, so $\frac{2}{8}$ must be removed to produce $\frac{7}{8}$. Since there are two fractions to be subtracted, $\frac{2}{8}$ can be written as $\frac{1}{8}$ twice in the spaces, fulfilling the conditions.

3. **(Missing information -- How much paper is in each pack?)** There is not enough information given to solve the problem.

4. **($404)** There are a number of ways that students might approach this problem. Akeem makes $8 x 40 = $320 for his regular work week. The remaining 7 hours is overtime, at "time and a half." Time and a half means $8 + $4 for each hour, instead of $8. So the overtime pay is $12 per hour. $12 x 7 = $84. $320 + $84 = $404.

5. **(Bachie -- 10 hrs, $57.50; Dustin -- 4.5 hrs, $25.88; Monica -- 8.5 hrs, $48.88)** Students can count from the In time to the Out time to find the hours worked. These hours might be written as a fraction also. Multiplying the hours worked by $5.75 gives the resulting amount earned. This is easy to do using a calculator if the hours are written as a decimal instead of as a fraction. Dustin's and Monica's "amounts earned" have been rounded up to the nearest cent.

6. **(9, 16, 100, *n*x*n* or n^2)** Students might be encouraged to build these figures out of cubes, and look at the pattern that occurs.

7. This problem encourages students to internalize what it means to *bisect an angle* before they meet the term later and are expected to use a compass to perform the task. They can hold the sheet of paper up to a light source, and fold the paper so that the two sides of the angle match up. Paper folding can also be used to teach terms such as *perpendicular bisector of a line segment.*

8. **(odd, even, yes)** Opening an actual book and looking at the page numbers will enable students to internalize what this problem means. Books universally start with page 1 on the right-hand side. The right-hand page numbers from there on, therefore, are always *odd.* The left-hand page numbers are always *even* numbers. An *odd* added to an *even* always gives an *odd* number. An *odd* times an *even* always produces an *even* number.

17

Commentary

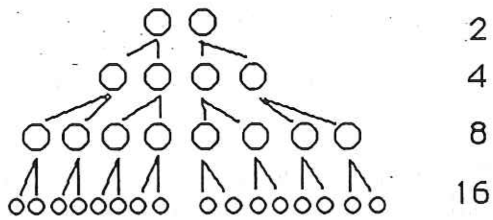
Saturn, XV

1. **(Ms. Hill - $10, Mr. Booth - $12.50)** Ms. Hill buys 50 shares for $500, and makes $0.20 on each for a total of $10. Mr. Booth buys 25 shares for $500, and makes $0.50 on each for a total of $12.50.

2. **(Tiffany has $20.50, Ivan has $0.50)** Subtracting $20 from $41 leaves $21 for Tiffany and Ivan together. Therefore, Ivan must have $0.50. Any other amount would mean Tiffany has more or less than $20 more than Ivan.

3. **(265)** When the students turn in their paper, have this problem on 3x5 cards for them to solve: *735 +* _____ *= 1000* Students MAY NOT use pencil, paper, or a calculator.

4. **(30 people)** A diagram such as the one to the right will help students realize that 30 have been invited altogether.



5. **(B)** $17 = 25 - p$ has the solution 8, which is the same solution as for $n + 13 = 21$. Students might be encouraged to think of the variable as representing a number they are searching for, to make the given statement a valid one. They can solve these by *guess-check-revise*.

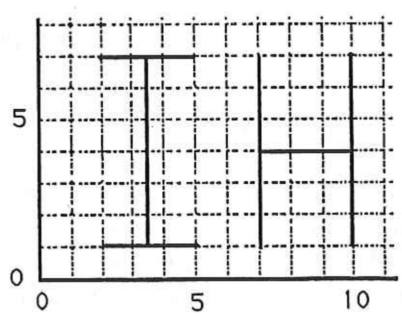
6. **(11)** Students might divide 238 by 23 and get 10 r 8, or add 23 until they get close to 238, counting the number of times they add. In any approach, 10 boxes hold 230 puzzles, with 8 puzzles left for the 11th box.

7. **($20.51)** One approach is to multiply $18.99 times 8%, rounding up to get the tax on the jacket. Add this tax to the price. A one-step approach is to multiply $18.99 by 1.08.

8. **(The answer is shown to the right.)** Students can use number clues to narrow down the choices for the box on the top row. Once that box is determined, the rest can be obtained by computation.

$$\begin{array}{r} 4\boxed{7} \\ \underline{\times 3\,5} \\ 2\boxed{3}5 \\ \underline{1\,4\,1\,0} \\ 1\boxed{6}4\boxed{5} \end{array}$$

9. **("HI" is spelled out, backwards)**



21

Commentary
Saturn, *XVI*

1. **(6.8 cm)** One approach is to add sides given and subtract the sum from the total perimeter. Another is to subtract each known side individually from the total.

2. **(See the worked-out problem to the right.)** Students who are new to this type of problem might begin at the end, and "work backward." (i.e., the box above 9 must have a 9 in it, which means that the box above it must be a 6 since that's what subtracts from 5 to get 9.) A few clever guesses and checks from this point will result in the solution.

$$\begin{array}{r} 27 \\ 13\overline{)351} \\ \underline{26} \\ 91 \\ \underline{91} \\ 0 \end{array}$$

3. **(2.50; 1.20; 130.0)** If students have not been taught to move the decimal point in divisor and dividend, they can think of these problems as explained below.

   a. 1.25÷ 0.5 means how many 1/2s are in 1 1/4. They can draw a diagram to find the answer is 2 1/2.

   b. There would have to be about one 7/10 in 8/10 because they're about the same size. So the only reasonable choice, of those given, is 1.20.

   c. They can think of (c) along these lines: there are 10 one-tenths in 1 whole, so there would be 13 x 10 one-tenths in 13. Therefore the answer is 130.

4. **(75)** The average would be found by adding 100, 90, 85, 78, 0, 80, and 92, and dividing by 7. Some students may mistakenly think that 0 doesn't count, and divide by 6 instead.

5. **(86)** You would follow the same steps as in number 4, but replace 0 with 70. Hopefully students will realize how important it is, if they are graded by an average, to not have a 0.

6. **(a. $2a$ - 3; b. $10h$ + 3; c. 3 + $\frac{1}{2}d$; d. $5x$ + 5.8)** Note that alternatives to the answers given should be accepted. For example, other common ways of writing (a) include $a + a - 3$ and $2Xa - 3$. Students need much practice with expressing these types of verbal situations mathematically, using variables.

7. **((a.) $\frac{26}{8}$ or $3\frac{2}{8}$ or $3\frac{1}{4}$ (b) 3)** Drawing a picture will help students count the 26 total blocks, and see that the total length is 26 X $\frac{1}{8}$ or $\frac{26}{8}$ miles. Their only task, then, is to express this amount as a fraction, mixed number, or possibly a decimal.

8. **(225)** Three $1\frac{1}{4}$'s is $3\frac{3}{4}$ total hours, which is 3 x 60 + 45 minutes.

9. **(B)** The chance of drawing a black marble is 3:5 for A, and 2:3 for B. Doubling and tripling, etc., these ratios until you find a "common unit" for the comparison is a good strategy. 3:5 is the same as 6:10, 9:15, etc. 2:3 is the same as 4:6, 6:9, 8:12, and 10:15. Notice we finally have a common unit -- 15 -- for comparison purposes. Since 10:15 is a higher ratio than 9:15, box B gives the best chance.

17 (+2) +19

Commentary
Saturn, *XVII*

1. **(604)** Have this problem -- $4\overline{)2416}$ -- prepared on 3x5 cards to give to students when they are about to hand in their paper. They can only write the answer, doing the division mentally.

2. **($3000)** $89 is about $90, and $90 x 4 weeks per month x 8 months = $2880, or $3000 to the nearest thousand.

3. **(131, 767, 505, 8998 are examples of correct answers)** Check to see if the numbers written are the same when the numbers are reversed, left to right.

4. **(This sentence is a palindrome made from letters instead of numbers.)** Some students who might not have seen the visual pattern in problem 3 will see the pattern here, and then can return to problem 3. Single words can be palindromes -- MOM, POP, BOB-- for example.

5. **(400,000,000)** Students can multiply 70 x 60 x 24 x 365 on a calculator, but the display will then be filled on an 8-digit model by 36,792,000. Multiplying this by 10 can be done mentally, giving 367,920,000. Rounded, the answer is 400,000,000.

6. **(60 and 30)** Students will need to realize that a right angle has 90°, and that the 2 equally spaced marks for 1:00 and 2:00 divide the 90° angle into three equal parts. The second hand pointing at 2:00 divides the 90° angle into 2/3 of 90° and 1/3 of 90°.

7. **(4, 2, 5)** The 13th digit will be the next one in line, and the pattern is ready to repeat again. Therefore the 13th and 14th digits will be 4 and 2. To find the 100th digit, notice the pattern repeats every 6 digits. 6 x 16 = 96, so the pattern will be ready to start repeating again with the 97th digit. The 97th digit is 4, the 98th is 2, the 99th is 8, and the 100th is 5.

8. **(One arrangement of the chart is shown below.)** The entries could be rearranged, but the sum of the numbers in each column must be 11, and the larger number must be on top.

Oranges	10	9	8	7	6
Watermelons	1	2	3	4	5

8. **($44.03)** Students may go through steps of finding the per cent and sales tax as separate processes, adding or subtracting them to the base figure they're working from. Or they might take a shortcut of taking 90% of the total cost of $46.15, and then multiplying by 1.06. On a calculator, this can become an easily done 2-step problem.

19

Commentary
Saturn, *XVIII*

1. **(a. 8; b. 6)** There is only a 2 in the circle for fish, meaning that there are 2 children with only fish for pets. There are 6 kids in the overlap area of dogs and fish, which means that 6 have both dogs and fish. So there are 8 altogether that have fish.

2. **(5.75)** The distance you ran before turning around, and the distance back to the starting point, was 3 miles. Then 2.75 is added to the first 3 miles.

3. **(19,000)** Multiplying 735 times 26 gives 19,110. Since this is only an approximation situation, rounding to the nearest thousand turns makes sense. 19,110 is closer to 19,000 than to 20,000, so it is rounded to 19,000.

4. **(2 hours, 45 minutes; 9:15; increasing)** The problem involves looking at a graph over time, and making judgements about the real-world situation from the shape of the curve. The curve stops at the 3rd "tick mark" after 10:00, and each tick mark stands for 15 minutes. So the race must end at 10:45, and lasted 2 hours 45 minutes. The rider stopped when the speed drops to zero, which is at 9:15. The line is steadily on the rise between 10:00 and 10:30, so his speed is increasing.

5. **(0.77)** There are 3 typical ways that students might make mistakes here. First, they might not "line up the decimal points" when adding the numbers -- if using a calculator, this won't be a problem. Second, they might divide by 4 instead of dividing by 5, forgetting to count Friday as the fifth day, with 0.00 inches. And last, they might try to "round off the answer", although it's unnecessary since it's already only to the hundredths place. The computation $(1.66 + 0.23 + 0.75 + 1.2) \div 5 = 0.77$ shows how to successfully solve the problem.

6. **(32)** Students can actually fold a sheet of paper to find the answer, if they don't have good visual skills.

7. **(18)** This can be thought of as a ratio problem. Three books every two weeks means 6 books every 4 weeks (or 1 month). Six books each month means 18 books for three months (or the summer).

8. **(3.3 and 3.8)** The line is divided into 10 parts. The arrows point to the 3rd and 8th *tick marks.* The answers can either be written as decimals (3.3 and 3.8) or mixed numbers ($3\frac{3}{10}$ and $3\frac{8}{10}$).

9. **(3 yards, 1 foot, 8 inches)** There are several ways to approach this problem. One way is to convert 10 yards, 2 feet into 384 inches, divide that by 3 to get 128 inches for each girl, then convert 128 inches into feet and yards by dividing first by 12, then by 3. Another, more challenging way is to solve the long-division problem below:

$$3\overline{\smash{)}\,10 \text{ yards } 2 \text{ feet } 0 \text{ inches}}$$

The challenge above is in renaming the number obtained after the subtraction step of the process, and joining it with the next digit.

Commentary

Saturn, XIX

1. **(1, 5, 14, 30)** Stuents can find these answer by actually counting squares. They are encouraged to do so in an organized fashion, starting with the smallest individual squares, then moving up to count larger squares.

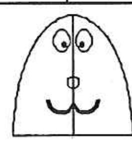
2. **(yes, 385)** The pattern does work for the next figure in line -- students can verify this by actually counting individual squares again. Assuming the pattern holds, the 10th figure would have $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 = 385$ squares.

3. **($15.89)** Multiply the price of the calculator times 6% and add that amount to the cost of the calculator. Or, you could do this all in one step on a calculator, by multiplying $14.99 x 1.06, rounding up the answer.

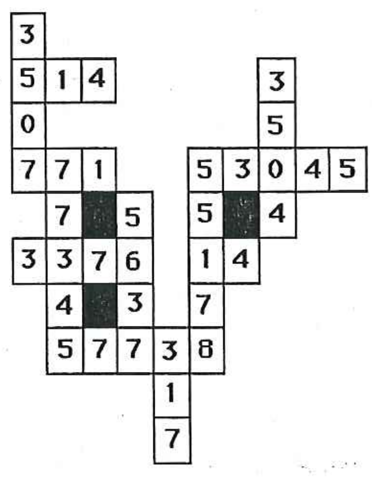
4. **(See the chart to the right.)**

		2	3	4	5
a.	6,945		✔		✔
b.	1,236,240	✔	✔	✔	✔
c.	54,208	✔		✔	

5. **(See drawing to the right.)**



6. **(The completed puzzle is shown to the right.)**



7. **(The completed problems are shown below.)** Look carefully at the partial products, as well as the answer. The partial products are where the differences show up.

14	31	27	62
x 26	x 53	x 42	x 135
104	53	294	270
260	1590	840	8100
364	1643	1134	8370

18

Commentary

Saturn, XX

1. **(28)** Encourage students to experiment with several other numbers less than 10, to check and see if they are *perfect numbers*. This will give them a feel for what they are searching for. The proper factors of 28 are 1, 2, 4, 7, and 14, and their sum is 28.

2. **(See problem to the right.)** Working backwards will help the students find the missing digits quickly.

$$\begin{array}{r} 317\text{ r }25 \\ 27\overline{)8584} \\ \underline{81} \\ 48 \\ \underline{27} \\ 214 \\ \underline{189} \\ 25 \end{array}$$

3. **(a. 364 yards b. 4 strokes c. 36 strokes)** Students unfamiliar with golf might profit from looking at a golf score card from a local course. Usually those cards have a picture of the course, with the yardage for each hole and the par for the hole. Looking at such a card and discussing the game in general -- how long a good drive might be, etc. --will help them understand the terms. The average distance per hole is found by computing 6550 ÷ 18; the average number of strokes per hole is given by 72 ÷ 18; Carlos was 108 - 72 or 36 strokes over par.

4. **(350 ml)** Students can simply look at a can of soda. They might also be encouraged to visualize a can of soda as about 1/3 liter.

5. **(122 oranges)** *Working backwards* is one strategy. Shomika had 26 oranges so that she could give Josie 1/2 of that, plus 3, leaving 10. Shomika had 58 oranges so she could give Angela half, plus 3, leaving 26. Shomika started with 122 oranges so she could give half, plus 3, to Jennifer, leaving 58. A different way to begin the problem is to *guess-check-revise*. You might begin by guessing 100, checking to see if Shomika winds up with 10 in the end. If not, revise the guess either higher or lower, depending on the result.

6. **(34)** Students should compute inside the parentheses first, then multiply before subtracting, following the *order of operations* rules.

7. **(a. 27 b. 63%)** There are 13 + 4 + 10 or 27 students. The diagram shows that 17 students like chocolate cupcakes. 17 out of 27 is the same as 17/27 or 17 ÷ 27, which is 63% when rounded to the nearest whole percent.

8. **(The fold line should show a line that forms 90^0 angles with the one given).** Students can find such a fold line by holding the paper up to a light source, and folding it so that the lines seen through the paper fall on top of each other. This problem can be expanded into finding the *perpendicular bisector* of a line segment. In this case, the endpoints of the segment would also have to line up, to be sure the fold cut the segment exactly in half.

9. **(2:30)** The boys worked five hours, and had a total of another hour of breaks. They were therefore at the clean-up site for 6 hours, starting at 8:30.

10. **(1 week, 5 days, 18 hours, 51 minutes)** This problem asks students to rename given units of time before subtracting. The value is that students will see that regrouping for subtraction requires them to think about the units involved.

+20

Commentary

Saturn, XXI

1. Mental math should help students solve this problem. Below is one possible set of answers. Accept other correct solutions.

4 - 2 X 1 = 2	**4 + 2 ÷ 1 = 6**
4 - 2 + 1 = 3	**2 + 1 + 4 = 7**
4 ÷ (2 - 1) = 4	**4 X 2 ÷ 1 = 8**
4 + 2 - 1 = 5	**4 X 2 + 1 = 9**

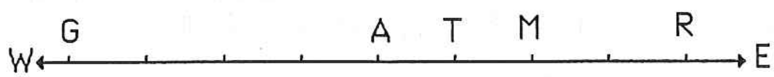
2. **(66 mi., $74.80)** If Brad can go 3 miles on each gallon, he can go 3 x 22 miles on 22 gallons. If each gallon costs $3.40, then 22 gallons cost $3.40 x 22.

3. **(a. 1,207 miles, b. 87,000 miles)** The nearest city is London, at 3,458 miles. The farthest is Moscow at 4665 miles. The difference is 4,665 – 3,458, or 1,207 miles. Round trip to Paris is 3,624 x 2; that distance 12 times is 86,976 miles, or about 87,000 miles.

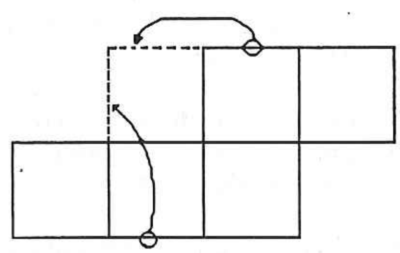
4. **(about 8,250,000 heads of lettuce)** Multiply 11,000 x 1,500 x 1/2. Drawing a picture of the acreage might benefit some students.

5. **(Harry: $5.62 and William: $3.37)** There are a number of ways to approach this problem. One is to divide $8.99 by 8, getting $1.12 per slice. Then multiplying by the number of slices will produce the answers. Another way is to realize that Harry ate $\frac{5}{8}$ and William $\frac{3}{8}$, convert each of those into a decimal or a percent, and multiplying each by $8.99 to get the answers.

6. **(See picture below.)** Students will use clues and measuring skills. Measure to be sure that the distances are correct. Answer below gives correct relational locations, but distances are approximations.



7. **(See picture below.)**



8. **($1.85)** Two pair for $1.98 means another pair can be purchased for half that, or $0.99. The total is then $2.97. Adding tax can be done by multiplying by 1.06 on a calculator, giving a total cost of $3.15. The change can be "counted up" from $3.15 to $5, getting $1.85.

9. **($\frac{28}{12}$ or $2\frac{4}{12}$ or $2\frac{1}{3}$)** Students can use the common denominator 12, convert each fraction to that denominator, and add or subtract the numerators. An interesting real-life problem for these numbers would be to use several 12-packs of colas to show the fractions.

x18

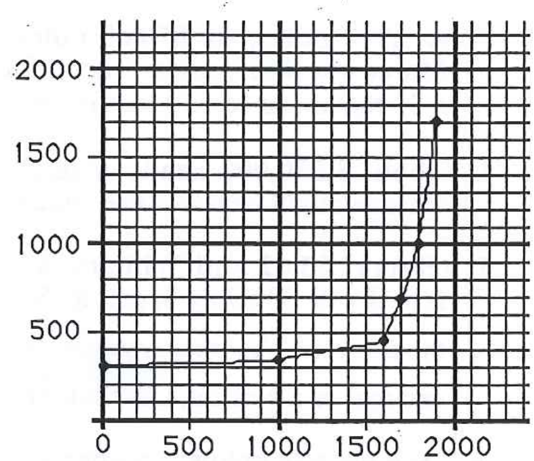
Commentary

Saturn, XXII

1. **(40)** Rather than trying to solve this equation through computation, students might think of solving it by asking "What number could *p* be so that so 3/4 of its weight is 30?" Intuition will lead most students to think of trying 40 for *p*, and asking "Is 3/4 of 40 equal to 30?" Yes, so their intuition is correct.

2. **(90 in^2)** Students might cut out a sheet of paper this size, and line it up repeatedly to approximate the area. Some students might think of marking off inches along the length and width, and computing with those numbers. A few might know that a sheet of paper like this is 8.5 by 11 inches, giving an actual area of 93.5 in^2.

3. **(Line graph shown to the right. Accept any answer over 2 billion for population in 2000 AD)** This problem might draw students into a discussion about the world's population and what will happen if it continues to grow unchecked.



4. **(124, 154, 300, 3 x *n* + 4)** It's difficult to tell what the machine is doing by looking at what happens to the first few numbers. A hint as to what the machine is doing (multiplying by 3, then adding 4) can be gained by looking at what happens to 10 and 100.

5. **(3,800)** If 3,400 is 90% of the number sought, then 3,400 can be divided by 0.90 and rounded to the nearest 100. Students may discover other ways to find this number, such as guessing what number from {3500; 3600; 3700; 3800; 3900; ...} might work, and checking to see if 90% of each gives 3400. Allow them to feel comfortable with their methods.

6. **(... so she would get back only one coin in change)** This is a common practice among people who have good number sense, and should be encouraged for students as it provides continuous practice with mental mathematics in a real-world setting.

7. **(4026 x 8)** One way to approach such a problem is to begin by seeking digits that would produce 32 in the thousands place of the answer, 4 and 8. From that point, *guessing and checking* will lead to the answer.

8. **(a. 11, 13, and 15; b. 99; c. 1999)** It's easy for students to go from one figure to the next, in progression. They will notice that these are the odd numbers in sequence. To find the number of dots in the 50th figure, some will actually count the odd numbers that far, while others will begin to approach it analytically -- the 50th odd number is one less than twice the number. This type of generalization is almost a necessity for obtaining the number of dots in the 1000 figure. An extension is to ask students, given the figure number *n*, what algebraic expression tells the number of dots needed?

16

Commentary

Saturn, XXIII

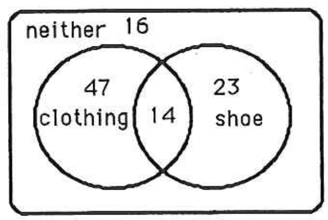
1. **($2.91)** The three girls' combined money is $7.04. Subtract this from the $9.95.

2. **($x = 26$; $y = 37$; $z = 9.2$)** These problems present solving simple equations in a way in which students can follow the logic of the steps typically involved. In the first, removing 2 from each side of the scale leaves the scale still balanced, and the cake weighing 26. This is called *isolating the variable.* In the second picture, intuitively you divide both sides of the scale by 3 to find what one clock weighs. In the last picture, if you remove 5 from the scale, 4 coins remain and weigh $41.8 - 5 = 36.8$. Then dividing by 4 means that each coin would weigh 9.2.

3. **(b. 30)** $13\frac{38}{39}$ and $7\frac{16}{17}$ both have fractions that are very close to 1, so these mixed numbers can be rounded to 14 and 8. $4\frac{1}{9}$ and $4\frac{1}{42}$ have fractions close to zero, and so each can be rounded down to 4. The estimated sum is then $14 + 8 + 4 + 4$ or 30. This is a good measure of *number sense.*

4. **(a. 40 sq. units)** In this problem, students can estimate the area of a figure visually. There are a number of ways to do this. One is to count whole and then partial squares, putting together partial ones to get whole ones. Another way is to count all the whole ones, and then just count the partial squares and divide it by two, which assumes that the partial squares will average about 1/2 sq unit each. There are many other ways to get the estimate.

5. **(See the diagram to the right.)** The difficult part for some students will be to remember to put in the diagram the number of people who liked neither store.

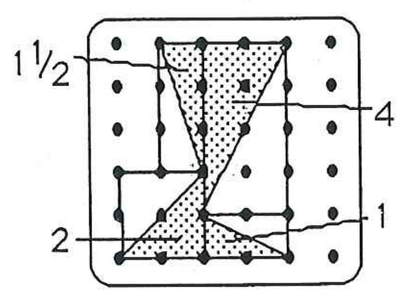


6. **(8,000 mi.)** This can be thought of in several ways. One approach is to reason as a ratio -- 2,000 miles in 1.5 minutes is 4,000 in 3 minutes, 6,000 in 4.5 minutes, and 8,000 in 6 minutes. Another approach, if a calculator is handy, is to find out with $2000 \div 1.5$ that the shuttle is travelling 1333.33 miles per minute; this number times six minutes would be 8,000.

7. **(Neither)** The probability of pulling out a penny is 1 out of 2, in both cases.

8. **($1.80)** Students can find 1/6 of 18 coins and know that there are 3 quarters, 1/3 of 18 is 6 dimes, and 1/2 of 18 is 9 nickels. These coins add to $1.80.

9. **(15, 9, $8\frac{1}{2}$)** These 3 figures get progressively more difficult to find the area by counting unit squares. The first and second, when grid lines are drawn in, can be found by counting whole squares and half squares. The third figure can be partitioned so that rectangles are drawn around certain trianglular parts. Since the area of the triangles are half of the surrounding rectangles, the partial areas can be found this way and summed for the total. One way to do this is shown.



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Commentary
Saturn, XXIV

1. **($1,010,000)** Students can perform arithmetic to find that 40,400 attended those two games. Students might enjoy investigating such large amounts of money for several sports events in large facilities in their state or across the country.

2. **(a. 100%, b. food, c. $2,400 d. $4,200)** (a) Students should recognize that the whole circle always represents 100% (b) Compare the percents for food and education. (c) Multiply 12% times $20,000. (d) Add 7%, 8%, and 6%, then find 21% of $20,000.

3. **(1:45)** Students with good spatial sense will likely get this answer immediately, if they realize that a mirror reverses images left-to-right. For students who have trouble visualizing the situation, they can actually hold the paper up to a mirror to see the time for themselves, or possibly hold the paper to a light source, turn it over, and look at the clock from the back side.

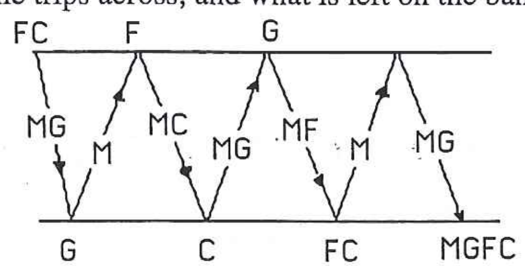
4. **(98.70 mph)** Round to the nearest hundredth.

5. **(a. 3 x x + 2, b. 100 -2 x s, c. $\frac{1}{2}$ x t -2)** The importance of students being able to rephrase real-world situations using a variable makes problems like this extremely valuable. It is unlikely at this stage that the students will have encountered the "shorthand notation" of writing 3 x x, 2 x s, and $\frac{1}{2}$ x t as $3x$, $2s$, and $\frac{1}{2}t$. It is unnecessary for them to use such conventions at this age.

6. **(a. 800)** Students can count the bricks along the length and width, and find that there are 36 rows with 21 in each row, for a total of 756 bricks. Notice that students can't count them all because of the lawn furniture and trees partially blocking the view. 800 would give some extras, but not so many as to cost a great deal more money. 700 is obviously too few.

7. **(b. 25°)** 25° C (or 77° F) is about room temperature. Most swimmers begin when the weather gets that warm. Students should develop benchmark temperatures in both Fahrenheit and Celsius -- such numbers as when water freezes and boils, their own body temperature, a cold drink, and so on.

8. (7) A drawing of the trips across, and what is left on the bank each time, is shown below:



+17

Commentary

Saturn, XXV

1. **(16 items -- 15 bats and 1 mitt or 14 bats and 2 mitts)** One approach is to make a chart of all of the combinations which could be purchased, and select the entry which gives the largest number of items. Another approach is to realize that the way to get the largest number of items is to purchase only one at the higher price, and spend the rest of the money on the lower priced items.

2. **(6/7)** Students can use *guess, check, revise* to find their answer. Number sense will tell them not to try fractions with the numerator bigger than the denominator, and that the numerator must be fairly close to the denominator in size.

3. **(a. $3.25 b.$1.75 c. see list below)**

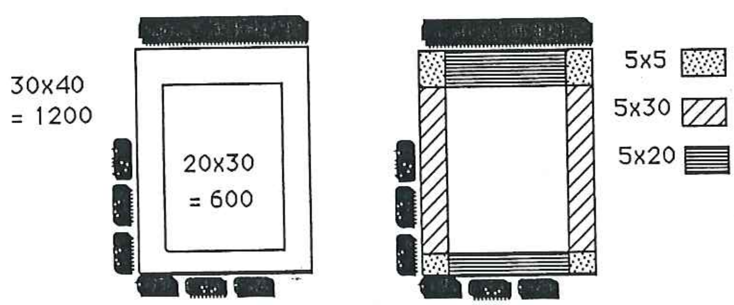
small fries, hot dog	*small fries, small shake*
small fries, small cola	*large fries, small cola*
small fries, large cola	*large fries, large cola*

4. **(325)** Elvira will have to multiply 13 by 5 to return to the previous stage in the problem, and multiply by 5 again to finish the problem correctly. The real answer, then, is 13 times 25 or 325.

5. **(a. the weight of one pup; b. the weight of all 6 pups is 6 x W ; c. 7 pounds)** This problem is designed for students to see a real-world use of algebraic equations, but one which they can solve by using *number sense*. At this point, they should attempt to find the value for W in any intuitive way that makes sense to them. Some will want to subtract 8 from 50, then divide by 6. This is the typical method that will be taught later and is fine if done intuitively. It's also acceptable for students at this point to simply search for a replacement value for W that makes sense. They can use *guess-check-revise* for this approach.

6. **(a. 30' by 40'; b. 600 ft^2)** Part (a) of the problem is easily solved by simply labelling the diagram of the pool, and adding on the extra 10 feet to each dimension. Students might approach part (b) in several ways. One is to calculate the area in square feet of the 30 by 40 pool and walk combined, and then subtract the area of the 20 by 30 pool itself. Another approach is to partition the concrete walk itself into smaller pieces, find the area of each, and add them together -- there are several ways to partition the walk in this fashion. Both approaches are shown below:

2



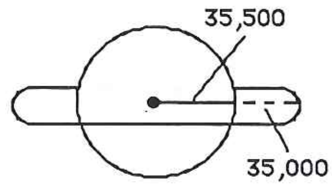
7. **(2 sections will be red, 4 will be blue, and 2 will be green.)** This problem should help students see a real-world example in which $\frac{1}{4} = \frac{2}{8}$ and $\frac{1}{2} = \frac{4}{8}$ is intuitively obvious. Visually they can see if they have $\frac{1}{4}$ or $\frac{1}{2}$ of the spinner shaded, so they can count the sections.

+21

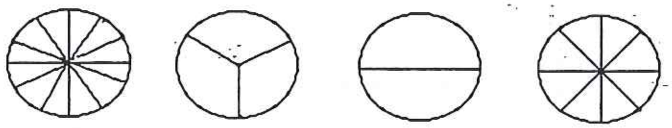
Commentary

Saturn, XXVI

1. **(70,500 miles)** Drawing a diagram will help students see that, to find the distance from the center to the outer edge of the rings, they need to find the radius of the planet, or 35,500 miles. This number is then added to the distance to the edge of the rings, 35,000.



2. **(The circles will appear as shown below.)** Students might try tracing the angles given using a dark pencil or ink pen, so the angle will show through from the underside of the paper. They can then use this dark angle to trace the angles in their circle.



3. **($\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$; 16 comes three numbers before 2)** Students should notice that each number of the pattern is one half the previous number. Making a rectangle and sketching in 1/4 will help them find 1/2 of 1/4, and then 1/2 of that, and so on. In this manner they will have a visual image to show that taking half of a fraction doubles the denominator. The pattern still holds, going to the left.

4. **(30 stamps)** Anne gave the first 50 stamps to Sam, leaving 75 stamps to divide. Each of the five friends would then receive 15 stamps.

5. **($\frac{12}{45}$, 20 tally marks, $\frac{20}{45}$)** The fraction $\frac{13}{45}$ combines the tally mark information with the number of spins, 45. In a similar fashion, the second fraction would then be $\frac{12}{45}$. The tally marks for green must then be 20 as they all sum to 45. The missing fraction is then $\frac{20}{45}$.

6. **(The completed problem is shown to the right.)** Students might find it easier to work backwards on this problem. If so, they can fill in the lower boxes first, simply by finding missing addends. Then number sense can take over and they can find the two missing numbers in the two numbers being multiplied.

$$\begin{array}{r} \mathbf{922} \\ \underline{\mathbf{x18}} \\ \mathbf{7376} \\ \underline{\mathbf{9220}} \\ \mathbf{16596} \end{array}$$

7. **(Susan is 15, Andrea is 5, and Barbara is 10)** Using *guess, check, revise* as a strategy, students may try various ages. It's probably easier to always start guessing with the youngest person's age, compute the others from that, and check against the conditions.

8. **(b. Slow down! ...)** Students will use different approaches in this problem. One obvious one is to divide 24 minutes by 3 miles and get that Andy needs to run each mile in 8 minutes. Therefore he needs to run 2 miles in 16 minutes, and he is running faster than that. Some students might have a difficult time accepting the strategy of slowing down in a race, if they have never run a long distance.

+19

Commentary

Saturn, XXVII

1. **(34)** The problem is not difficult to solve, except that some students will forget to count Brandon himself.

2. **(d)** There are 14 coins, 7 of which are pennies. Therefore a penny will be drawn about half the time. So out of 10 draws, you would expect to get 5 pennies. Students might enjoy testing this theory by actually drawing coins at random.

3. **(See one solution to the right.)** This computational puzzle is one which has a number of different solutions. Students will enjoy coming back to this puzzle throughout the year, and will usually get a different answer each time. One hint which might get students started is to realize that 10 must go in the top row, because it can't be the difference of two other numbers.

   6 10 1 8

   4 9 7

   5 2

   3

4. **(0, 1, 2, 3, 4)** Students can intuitively see that *y* must be a whole number less than 5. Some students might not include 0 in the solution because the physical situation -- an object on a scale -- would naturally have some weight. Give them credit if they do not include 0, but be sure they understand that some things (Styrofoam and plastic wrap used in grocery stores to wrap meat, for example) have such a miniscule weight, that they are counted as 0.

5. **(125; 250; 50; 75)** There are a number of ways students can use *number sense* to find 1/4 of 500, 1/2 of 500, and 1/10 of 500. One hint for a student having trouble is to think of the 500 as 500 pennies, or $5. Once they determine the first three fractional parts of 500, they can then find the remaining number -- the cups -- by subtraction.

6. **(28 mm; 10 m; 3 km; 30 m)** Students should be encouraged to remember benchmark measures for distances in the metric system, and use those for estimation purposes. They might remember something that is a millimeter long, a centimeter long, a decimeter long, a meter long, and a kilometer long.

7. **($22.50)** Students should use the form of a percent that makes sense for a given problem. In this case, knowing 25% is equivalent to 1/4 might enable them to easily find one week's savings, as they can divide $10 into 4 equal shares. $2.50 a week for 9 weeks is $22.50.

8. **(a. 16 b. 26 c. 4 d. 4 to 140)** For students who are new to Venn diagrams, it would be helpful for them to start with some simple ones -- limited to 2 circles -- to understand what the various numbers mean, in overlap areas. They can gradually increase the difficulty.